Exercise 17.1

Q. 1. (i) $x^2 = 21^2 + 20^2$

$x = 29$

(ii) $x^2 = 13^2 + 84^2$

$x^2 = 7{,}225$

$x = 85$

(iii) $x^2 + 4^2 = 5^2$

$x^2 = 9$

$x = 3$

(iv) $x^2 + 10^2 = 26^2$

$x^2 = 576$

$x = 24$

(v) $25^2 = x^2 + 7^2$

$x^2 = 576$

$x = 24$

(vi) $41^2 - 9^2 = x^2$

$x = 40$

Q. 2. (i) $x^2 = 3^2 + 1^2$

$x^2 = 10$

$x = \sqrt{10}$

(ii) $x^2 = 1^2 + 1^2$

$x^2 = 2$

$x = \sqrt{2}$

(iii) $x^2 = 3^2 + 5^2$

$x^2 = 34$

$x = \sqrt{34}$

(iv) $7^2 = 5^2 + x^2$

$x^2 = 24$

$x = \sqrt{24}$

Q. 3.

$x = \sqrt{6.5^2 - 6^2}$

$= 2.5$ m

6.5 m

6 m

x
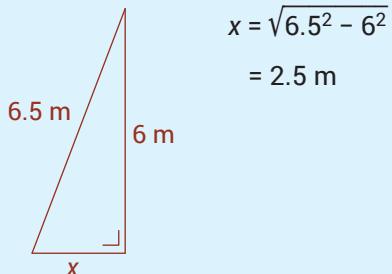
Q. 4. (i) $x^2 = 4^2 + 3^2$

$x = 5$

$y^2 + 5^2 = 13^2$

$y^2 = 13^2 - 5^2$

$y^2 = 144$

$y = 12$

(ii) $13^2 + 84^2 = x^2$

$x = 85$

$85^2 = 36^2 + y^2$

$y^2 = 5{,}929$

$y = 77$

(iii) $100^2 = 28^2 + x^2$

$x = 96$

$800 - 28^2 = y^2$

$16 = y^2$

$y = 4$

(iv) $x^2 = 8^2 + 6^2$

$x^2 = 100$

$x = 10$

$26^2 = 10^2 + y^2$

$y^2 = 576$

$y = 24$

Q. 5.

80

60
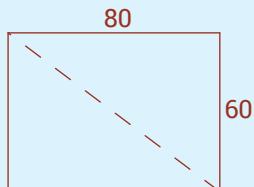
(i) $280 - 2(80) = 120$

$120 \div 2 = 60$ cm

(ii) $x^2 = 80^2 + 60^2$

$x^2 = 10{,}000$

$x = 100$

(iii) Area $= 80 \times 60 = 4{,}800$ cm^2

Q. 6. Triangle with sides: 85, 77, 36

$85^2 = 7{,}225$, $77^2 + 36^2 = 5{,}929 + 1{,}296 = 7{,}225$

$\Rightarrow$ Triangle is right-angled.

Q. 7. Triangle with sides: 7, 24, 25

$25^2 = 625$, $7^2 + 24^2 = 49 + 576 = 625$

$\Rightarrow$ Triangle is right-angled.

Q. 8. Triangle with sides 11, 60, 62

$62^2 = 3{,}844$, $11^2 + 60^2$
$= 121 + 3{,}600 = 3{,}721$

$\Rightarrow$ Triangle is not right-angled.

Q. 9. The width of a door frame is 84 cm and its height is 187 cm

84 cm

X   187 cm
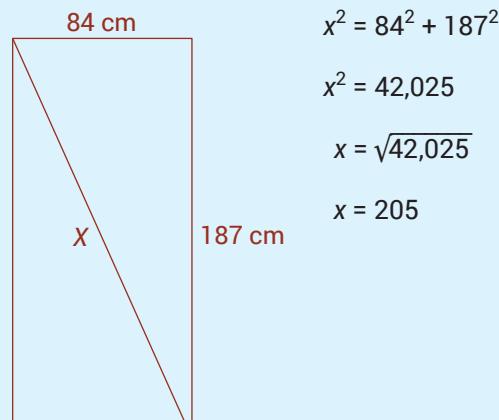
$x^2 = 84^2 + 187^2$

$x^2 = 42{,}025$

$x = \sqrt{42{,}025}$

$x = 205$

Exercise 17.2

Q. 1. (i) $\sin A = \dfrac{5}{13}$ $\cos A = \dfrac{12}{13}$ $\tan A = \dfrac{5}{12}$

(ii) $\sin A = \dfrac{21}{29}$ $\cos A = \dfrac{20}{29}$ $\tan A = \dfrac{21}{20}$

Q. 2. (i) $\sin A = \dfrac{40}{58}$ $\cos A = \dfrac{42}{58}$ $\tan A = \dfrac{40}{42}$ $\sin B = \dfrac{42}{58}$ $\cos B = \dfrac{40}{58}$ $\tan B = \dfrac{42}{40}$

(ii) $\sin A = \dfrac{3}{\sqrt{13}}$ $\cos A = \dfrac{2}{\sqrt{13}}$ $\tan A = \dfrac{3}{2}$ $\sin B = \dfrac{2}{\sqrt{13}}$ $\cos B = \dfrac{3}{\sqrt{13}}$ $\tan B = \dfrac{2}{3}$

(iii) $\sin A = \dfrac{2}{\sqrt{20}}$ $\cos A = \dfrac{4}{\sqrt{20}}$ $\tan A = \dfrac{2}{4}$ $\sin B = \dfrac{4}{\sqrt{20}}$ $\cos B = \dfrac{2}{\sqrt{20}}$ $\tan B = \dfrac{4}{2}$

Q. 3. (i) 0.2588   (v) 0.9004   (ix) 5.6713

(ii) 0.8660   (vi) 0.0872   (x) 0.4791

(iii) 3.7321   (vii) 0.2126

(iv) 0.2419   (viii) 0.5000

Q. 4. (i) 38.26   (vii) 58.54

(ii) 29.61   (viii) 46.23

(iii) 19.76

(iv) 25.94

(v) 20.62

(vi) 82.84

Q. 5. $\tan B = \dfrac{9}{40}$

$B = \tan^{-1}\left(\dfrac{9}{40}\right)$

$= 12.6804$

$\cong 13°$

Q. 6. $\cos C = \dfrac{15}{17}$

$= 28.0725$

$\cong 28°$

Q. 7. $\sin A = \dfrac{4}{5}$

$\therefore A = \sin^{-1}\dfrac{4}{5}$

$= 53°$

Q. 8. $\tan \theta = \dfrac{32}{63}$

$\therefore \theta = \tan^{-1}\dfrac{32}{63}$

$\theta = 27°$

Q. 9. The smallest angle is opposite the smallest side.

$\tan \theta = \dfrac{7}{24}$

$\theta = \tan^{-1} \dfrac{7}{24}$

$\theta = 16.3°$

Exercise 17.3

Q. 1. (i) $\dfrac{x}{5} = \tan 60°$

$x \cong 8.66$

(ii) $\dfrac{x}{10} = \cos 60°$

$x = 10 \cos 60°$

$x = 5$

(iii) $\dfrac{x}{18} = \cos 55$

$x = 10.32$

(iv) $\dfrac{x}{20} = \sin 35$

$x = 10.45$

(v) $\dfrac{x}{16} = \sin 30$

$x = 8$

(vi) $\dfrac{x}{40} = \cos 45$

$x = 30.42$

Q. 2. (i) $y = 7 \tan 32$

$= 4.37$

(ii) $y = 50 \sin 29$

$= 24.24$

(iii) $y = 18 \cos 35$

$= 14.74$

Q. 3. (i) $\sin 25 = \dfrac{11}{y}$

$\therefore (\sin 25)y = 11$

$y = \dfrac{11}{\sin 25}$

$= 26.03$

(ii) $\cos 50° = \dfrac{y}{20}$

$\therefore y = 20 \cos 50$

$y = 12.86$

(iii) $\tan 48 = \dfrac{10}{y}$

$\therefore (\tan 48)\,y = 10$

$y = \dfrac{10}{\tan 48}$

$= 9.00$

Q. 4. (i) $\sin A = \dfrac{4}{7}$

$A = \sin^{-1} \dfrac{4}{7}$

$A = 34.85°$

$A = 35°$

(ii) $\cos A = \dfrac{2}{6}$

$A = \cos^{-1} \dfrac{2}{6}$

$A = 70.52°$

$A = 71°$

(iii) $\tan A = \dfrac{5}{6}$

$A = \tan^{-1} \dfrac{5}{6}$

$A = 39.8°$

$A = 40°$

(iv) $\tan A = \dfrac{5}{3}$

$A = \tan^{-1} \dfrac{5}{3}$

$A = 59.03°$

$A = 59°$

(v) $\sin A = \dfrac{7}{8}$

$A = \sin^{-1} \dfrac{7}{8}$

$A = 61.04°$

$A = 61°$

(vi) $\tan A = \dfrac{4}{4}$

$A = \tan^{-1} 1$

$A = 45°$

Q. 5. $x = 100 \sin 40°$

$= 64.28$

$y = 64.28 \tan 10°$

$y = 11.33$

Q. 6. $x = 3 \tan 50°$ $z^2 = 13^2 - 4.7^2$

$= 3.5753$ $z^2 = 146.91$

$\cong 3.6$ $z \cong 12$

$y^2 = 3.6^2 + 3^2$

$y = \sqrt{21.96} \cong 4.7$

Exercise 17.4

Q. 1. $\dfrac{x}{103.7 \text{ m}} = \tan 77°$

$x = 103.7 \tan 77$

$x = 449$ m

Q. 2. $\dfrac{200}{x} = \tan 30°$

$200 = x \tan 30°$

$x = \dfrac{200}{\tan 30°}$

$x = 346$ m

Q. 3. (i) $\sin 47° = \dfrac{x}{22}$

$x = 22 \sin 47$

$\cong 16$ metres

(ii) $\dfrac{16}{x} = \sin 63°$

$x = \dfrac{16}{\sin 63°}$

$x = 18$ m

Q. 4. (i) $\cos 67 = \dfrac{46}{x}$

$x = \dfrac{46}{\cos 67}$

$x = 118$ km

(ii) $\dfrac{46}{2} = 23$ km/hr

(iii) $\dfrac{118}{2} = 59$ km/hr

Q. 5. $\tan 33° = \dfrac{x}{5}$

$x = 5 \tan 33°$

$= 3.25$

Liam's height 1.7 m

Tree $\quad$ 1.7 + 3.25 = 4.95 metres

Q. 6. $\tan 78° = \dfrac{x}{100}$

$x = 100 \tan 78°$

$= 470$ m

Q. 7. (i)
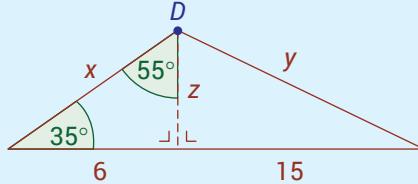
find x

$\cos 35 = \dfrac{6}{x}$

$\cos 35 = 0.81915$

$0.81915 = \dfrac{6}{x}$

$(0.81915)(x) = 6$

$x = 7.3246$ m = 7.32 m

Multiplied by 2

$= 14.64$ m

Total length of wire = (15.58 + 7.32) × 2

$= 45.8$ m

(ii) $\angle ADB = 90 - 35 = 55°$

$\angle ADC = \tan^{-1}\dfrac{15}{4.2}$

$\angle ADC = 74.36°$

find y

$(7.3246)^2 = (6)^2 + z^2$

$z^2 = 17.6497$

$z = 4.2$

$(4.2)^2 + (15)^2 = (y)^2$

$(y)^2 = 242.64$

$y = 15.5769$ m = 15.58 m

Multiplied by 2

$= 31.16$

Q. 8.
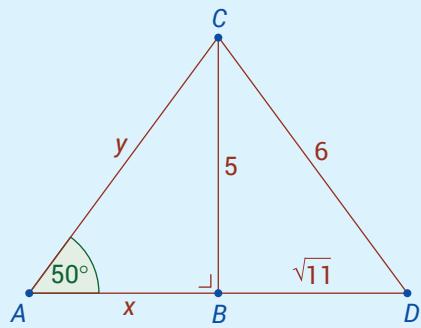
(i) $|BD|^2 = 6^2 - 5^2$

$|BD|^2 = 36 - 25$

$|BD|^2 = 11$

$|BD| = \sqrt{11}$

$|BD| = 3.3166$ m

$|BD| = 332$ cm

(ii) $\sin(\angle CDB) = \dfrac{5}{6}$

$\angle CDB = \sin^{-1}\dfrac{5}{6}$

$\angle CDB = 56.44°$

$\angle CDB = 56°$

(iii) **find x**

$\tan 50 = \dfrac{5}{x}$

$\tan 50 = 1.1917$

$\dfrac{5}{x} = 1.1917$

$(1.1917)(x) = 5$

$x = 4.195$

$x = 4.2$ m

find y

$\sin 50 = \dfrac{5}{y}$

$\sin 50 = 0.766$

$\dfrac{5}{y} = 0.766$

$(0.766)(y) = 5$

$y = 6.527$

$y = 6.53$ m

Total length $= 4.2 + 6.53 + 6 + 3.3$

$\qquad = 20.03$

+ 5% wastage

$\dfrac{20.03}{100} \times 105 = 21.03$ m

Q. 9.

(i) 30°

(ii) $\tan 60° = \dfrac{|AB|}{1}$

$\sqrt{3} = |AB|$

(iii) $1^2 + \left(\sqrt{3}\right)^2 = |AC|^2$

$1 + 3 = |AC|^2$

$|AC| = \sqrt{4}$

$|AC| = 2$

Q. 10.

(i) $180 - 90 - 45 = 45°$

(ii) $\cos 45° = \dfrac{|BC|}{7\sqrt{2}}$

$\dfrac{1}{\sqrt{2}} = \dfrac{|BC|}{7\sqrt{2}}$

$|BC| = \dfrac{7\sqrt{2}}{\sqrt{2}}$

$|BC| = 7$

Q. 11.

(i) $\cos 45° = \dfrac{x}{4\sqrt{2}}$

$x = 4$

Isoceles Δ so $y = 4$.

(ii) 45°

Q. 12.

(a) $\sin 60° = \dfrac{|AB|}{10}$

$\therefore |AB| = 10 \sin 60°$

$\qquad = 8.66$ cm

(b) (i) $\tan 65° = \dfrac{|OP|}{10}$

$\therefore |OP| = 10 \tan 65$

$\qquad = 21.4$ cm

(ii) $|\angle NOP| = 180° - (65° + 90°)$

$\qquad = 25°$

$\cos(\angle NOP) = \dfrac{|OP|}{|ON|}$

$\qquad = \dfrac{21.4}{30}$

$\therefore |\angle NOP| = \cos^{-1}\left(\dfrac{21.4}{30}\right)$

$\qquad = 44.5°$

$|\angle MON| = |\angle MOP| + |\angle MOP|$

$\qquad = 25° + 44.5°$

$\qquad = 69.5°$

(c) (i)

$$\tan 68° = \frac{30}{|AB|}$$

$$\therefore |AB| \tan 68° = 30$$

$$|AB| = \frac{30}{\tan 68°} = 12.12 \text{ m}$$

$$\tan 33° = \frac{30}{|AC|}$$

$$|AC| \tan 33° = 30$$

$$|AC| = \frac{30}{\tan 33}$$

$$|AC| = 46.20 \text{ m}$$

$$\therefore |BC| = 46.20 - 12.12 = 34.08 \text{ m}$$
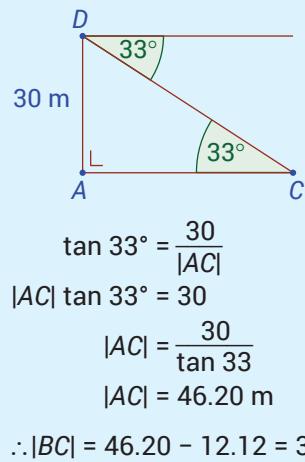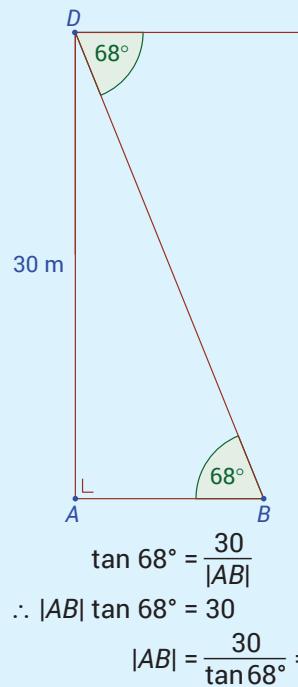
(ii) Average speed $= \dfrac{\text{distance}}{\text{time}}$

$$= \frac{34.08}{10}$$

$$= 3.4 \text{ ms}^{-1}$$

Exercise 17.5

Q. 1. $\frac{1}{2}(12)(15) \sin 55° = 73.72$ sq units

Q. 2. $\frac{1}{2}(24)(20) \sin 30° = 120$ sq units

Q. 3. $\frac{1}{2}(14)(18) \sin 40° = 80.99$ sq units

Q. 4. $\frac{1}{2}(25)(30) \sin 25° = 158.48$ sq units

Q. 5. $\frac{1}{2}(10)(10) \sin 70° = 46.98$ sq units

Q. 6. $\frac{1}{2}(7)(8.4) \sin 62° = 26$ cm^2

Q. 7. $\frac{1}{2}(9)(18.4) \sin 82° = 83$ cm^2

Q. 8. (i)
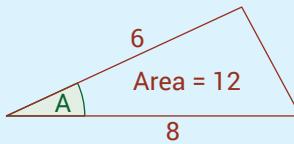
$$\text{Area} = \frac{1}{2} ab \sin C$$

$$12 = \frac{1}{2}(6)(8) \sin A$$

$$\sin A = \frac{1}{2}$$

$$A = \sin^{-1} \frac{1}{2}$$

$$A = 30°$$

(ii)
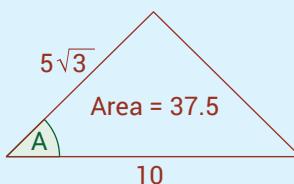
$$\text{Area} = \frac{1}{2} ab \sin C$$

$$37.5 = \frac{1}{2} 5(\sqrt{3})(10) \sin A$$

$$\sin A = \frac{\sqrt{3}}{2}$$

$$A = \sin^{-1} \frac{\sqrt{3}}{2}$$

$$A = 60°$$

Q. 9. (i)
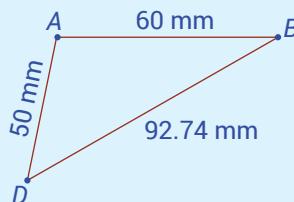
$$\frac{1}{2} \times 60 \times 92.74 \times \sin(\angle ABD) = \frac{2,227.18}{2}$$

$$2,782.2 \sin(\angle ABD) = 1,113.59$$

$$\sin(\angle ABD) = \frac{1,113.59}{2,782.2}$$

$$\therefore |\angle ABD| = \sin^{-1}\left(\frac{1,113.59}{2,782.2}\right)$$

$$= 23.59°$$
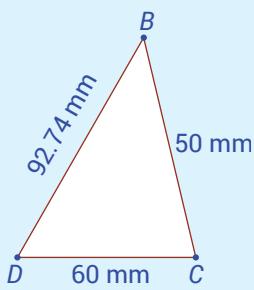
$$\frac{1}{2} \times 50 \times 92.74 \times \sin(\angle DBC) = \frac{2{,}227.18}{2}$$

$$2{,}318.5 \sin(\angle DBC) = 1{,}113.59$$

$$\sin(\angle DBC) = \frac{1{,}113.59}{2{,}318.5}$$

$$\therefore |\angle DBC| = \sin^{-1}\frac{1{,}113.59}{2{,}318.5}$$

$$|\angle DBC| = 28.71°$$

(ii) $|\angle DAB| = 180° - (23.59° + 28.71°)$

$$= 127.7°$$

Q. 10.   (i) $\cos \angle XYZ = \dfrac{3}{5}$
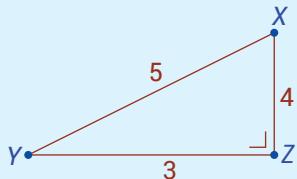
$$(XZ)^2 = 5^2 - 3^2$$

$$(XZ)^2 = 16$$

$$(XZ) = 4$$

$$\sin \angle XYZ = \frac{4}{5}$$

(ii) Area $= \dfrac{1}{2}\, ab \sin C$

$$\text{Area} = \frac{1}{2}(6)(4)\left(\frac{4}{5}\right)$$

$$\text{Area} = \frac{48}{5}$$

$$\text{Area} = 9.6 \text{ units}^2$$

Exercise 17.6
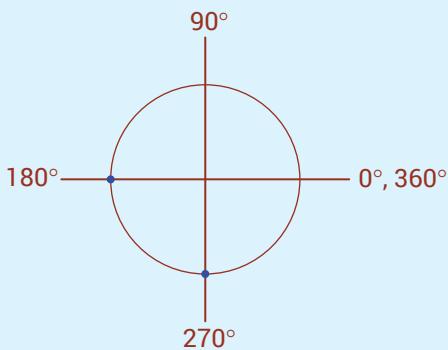
Q. 1.   0

Q. 2.   −1

Q. 3.   0

Q. 4.   1

Q. 5.   0

Q. 6.   −1

Q. 7.   0

Q. 8.   1

Q. 9.   0

Q. 10.   0

Q. 11.   1

Q. 12.   0

Q. 13.   0

Q. 14.   cos 135°   180 − 135° = 45°

−cos 45°

$$= -\frac{1}{\sqrt{2}}$$

Q. 15.   sin 150°   180 − 150° = 30°

sin 30°

$$= \frac{1}{2}$$

Q. 16.   cos 240°   240° − 180° = 60°

−cos 60°

$$= -\frac{1}{2}$$

Q. 17. sin 330°   360° − 330° = 30°

−sin 30°

$= -\dfrac{1}{2}$

Q. 18. tan 210°   210° − 180° = 30°

tan 30°

$= \dfrac{1}{\sqrt{3}}$

Q. 19. cos 315°   360° − 315° = 45°

cos 45°

$= \dfrac{1}{\sqrt{2}}$

Q. 20. sin 120°   180° − 120° = 60°

sin 60°

$= \dfrac{\sqrt{3}}{2}$

Q. 21. cos 210°   210° − 180° = 30°

−cos 30°

$= -\dfrac{\sqrt{3}}{2}$

Q. 22. tan 300°   360° − 300° = 60°

−tan 60°

$= -\sqrt{3}$

Q. 23. $\tan 60° = \sqrt{3}$

Q. 24. −0.82

Q. 25. 0.34

Q. 26. −0.64

Q. 27. −0.09

Q. 28. 0.84

Q. 29. 0.82

Q. 30. 0.64

Q. 31. −0.64

Q. 32. −0.18

Q. 33. −0.36

Q. 34. $\sin A = \dfrac{1}{2}$ $0 \leqslant A \leqslant 360°$

sin is positive

A = 30°   **OR**   A = 180 − 30°

A = 150°

Q. 35. $\cos A = \dfrac{1}{\sqrt{2}}$ $0 \leqslant A \leqslant 360$

cosine is positive

A = 45°   **OR**   A = 360° − 45°

A = 315°

Q. 36. $\tan A = \sqrt{3}$ $0 \leqslant A \leqslant 360$

60°	60°
60°	60°

S	A
T	C

tan is positive
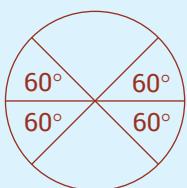
$A = 60°$ **OR** $A = 180° + 60°$
$A = 240°$
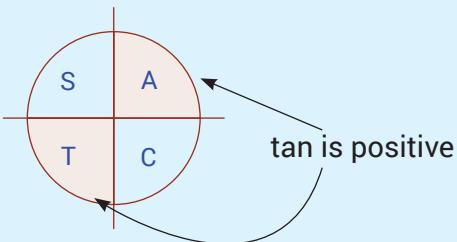
Exercise 17.7

Q. 1. $\dfrac{x}{\sin 50°} = \dfrac{10}{\sin 40°}$

$x = \dfrac{10 \sin 50°}{\sin 40°}$

$x = 11.92$

Q. 2. $\dfrac{x}{\sin 30°} = \dfrac{20}{\sin 40°}$

$x = \dfrac{20 \sin 30°}{\sin 40°}$

$x = 15.56$

Q. 3. $\dfrac{x}{\sin 125°} = \dfrac{10}{\sin 25°}$

$x = \dfrac{10 \sin 125°}{\sin 25°}$

$x = 19.38$

Q. 4. $\dfrac{x}{\sin 30°} = \dfrac{20}{\sin 100°}$

$x = \dfrac{20 \sin 30°}{\sin 100°}$

$x = 10.15$

Q. 5. $\dfrac{x}{\sin 70°} = \dfrac{10}{\sin 60°}$

$x = \dfrac{10 \sin 70°}{\sin 60°}$

$x = 10.85$

Q. 6. $\dfrac{\sin A}{12} = \dfrac{\sin 140°}{25}$

$\sin A = \dfrac{12 \sin 140°}{25}$

$A = \sin^{-1}\left(\dfrac{12 \sin 140°}{25}\right)$

$A = 18°$

Q. 7. $\dfrac{\sin A}{16} = \dfrac{\sin 44.5°}{14}$

$\sin A = \dfrac{16 \sin 44.5°}{14}$

$A = \sin^{-1}\left(\dfrac{16 \sin 44.5°}{14}\right)$

$A = 53°$

Q. 8.
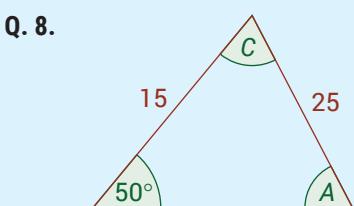
(i) $\dfrac{\sin A}{15} = \dfrac{\sin 50}{25}$

$\sin A = .45962$

$A = \sin^{-1} .45962$

$A = 27°$

(ii) $C = 180 - (50 + 27)$

$C = 103$

(iii) Area $= \dfrac{1}{2} ab \sin C$

Area $= \dfrac{1}{2}(15)(25)(\sin 103)$

Area $= 182.7$ units^2

Q. 9.
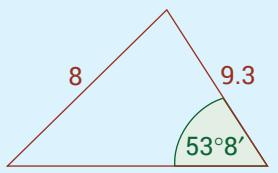
(i) $\dfrac{\sin A}{9.3} = \dfrac{\sin (53°\ 8')}{8}$

$\sin A = .93$

$A = \sin^{-1} .93$

$A = 68.44°$

$A = 68°$

(ii) $180 = C + 68 + 53$

$C = 59°$

(iii) Area $= \dfrac{1}{2}ab \sin C$

Area $= \dfrac{1}{2}(8)(9.3)(\sin 59°)$

Area $= 31.9$ units^2

(iv) Area $= \dfrac{1}{2}ab \sin C$

$31.9 = \dfrac{1}{2}(8)(C)(\sin 68)$

$C = 8.60$

Q. 10.
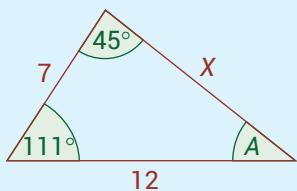
(i) $\dfrac{\sin A}{7} = \dfrac{\sin 45}{12}$

$\sin A = \dfrac{7\sqrt{2}}{24}$

$A = 24.36°$

$A = 24.4°$

(ii) Area $= \dfrac{1}{2}ab \sin C$

Area $= \dfrac{1}{2}(7)(12)(\sin 111)$

Area $= 39.2$ units^2

(iii) Area $= \dfrac{1}{2}ab \sin C$

$39.2 = \dfrac{1}{2}(12)(x)(\sin 24)$

$x = 16.06$

Q. 11. $\dfrac{|PQ|}{\sin 60°} = \dfrac{80}{\sin 70°}$

$|PQ| = \dfrac{80 \sin 60°}{\sin 70°}$

$= 73.73$ m

$\dfrac{|PR|}{\sin 50°} = \dfrac{80}{\sin 70°}$

$|PR| = \dfrac{80 \sin 50°}{\sin 70°}$

$= 65.22$ m

Q. 12.
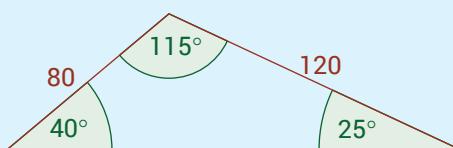
$\dfrac{\sin x}{80} = \dfrac{\sin 40°}{120}$

$\sin x = \dfrac{80 \sin 40°}{120}$

$x = \sin^{-1}\left(\dfrac{80 \sin 40°}{120}\right)$

$x = 25°$

Area $= \dfrac{1}{2}(80)(120) \sin 115°$

$= 4{,}350$ metres^2

Exercise 17.8

Q. 1. $a^2 = 4^2 + 5^2 - 2(4)(5) \cos 60°$

$a^2 = 21$

$a = \sqrt{21} = 4.58$

Q. 2. $a^2 = 5^2 + 10^2 - 2(5)(10) \cos 70°$

$a^2 = 90.7980$

$a = 9.53$

Q. 3. $a^2 = 9^2 + 12^2 - 2(9)(12) \cos 80°$

$a^2 = 187.4920$

$a = 13.69$

Q. 4. $a^2 = 5^2 + 5^2 - 2(5)(5) \cos 45.5°$

$a^2 = 14.9545$

$a = 3.87$

Q. 5. $a^2 = 15^2 + 12^2 - 2(15)(12) \cos 85°$

$a^2 = 337.6239$

$a = 18.37$

Q. 6. $\cos A = \dfrac{5^2 + 8^2 - 7^2}{2(5)(8)}$

$A = \cos^{-1}(0.5)$

$= 60°$

Q. 7. $\cos A = \dfrac{5^2 + 3^2 - 7^2}{2(5)(3)}$

$A = \cos^{-1}(-0.5)$

$A = 120°$

Q. 8. $\cos A = \dfrac{5^2 + 6^2 - 4^2}{2(5)(6)}$

$A = \cos^{-1}(0.75)$

$A = 41°$

Q. 9. $\cos A = \dfrac{10^2 + 9^2 - 3^2}{2(10)(9)}$

$A = \cos^{-1}(0.9556)$

$A = 17°$

Q. 10. (a) (i) $|AC|^2 = 4^2 + 6^2 - 2(4)(6) \cos 60°$

$= 28$

$|AC| = \sqrt{28} = 5.29$

(ii)
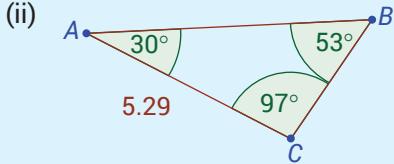
$\dfrac{|BC|}{\sin 30°} = \dfrac{5.29}{\sin 53°}$

$|BC| = \dfrac{5.29 \sin 30°}{\sin 53°}$

$|BC| = 3.31$

(b) Area of $ABC = \dfrac{1}{2}(52.9)(33.1) \sin 97°$

$= 869 \text{ mm}^2$

Area of $ADC = \dfrac{1}{2}(40)(60) \sin 60°$

$= 1039 \text{ mm}^2$

Area of $ABCD = 1908 \text{ mm}^2$

Q. 11. (i)

(ii) 96°

(iii)
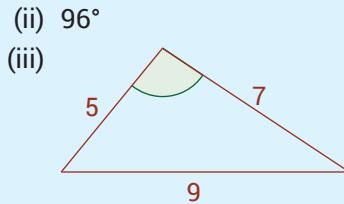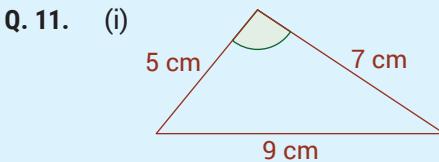
$\cos x = \dfrac{5^2 + 7^2 - 9^2}{2(5)(7)}$

$x = \cos^{-1}(-0.1)$

$x = 96°$

(iv) $\dfrac{1}{2}(5)(7) \sin 96°$

17.40 cm^2

Q. 12. (i)

(ii) 22°

(iii)
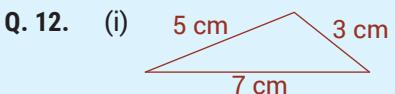
$\cos x = \dfrac{5^2 + 7^2 - 3^2}{2(5)(7)}$

$x = \cos^{-1}(0.9286)$

$x = 22°$

(iv) $\dfrac{1}{2}(5)(7) \sin 22°$

6.5556 cm^2

Q. 13. (i) $60 \times 0.25 = 15 \text{ km}$

(ii) $50 \times 0.25 = 12.5 \text{ km}$

(iii) 150°

(iv) $x^2 = 15^2 + 12.5^2 - 2(15)(12.5) \cos 150°$

$= 706.0095$

$x = 26.571 \text{ km} = 26{,}571 \text{ m}$

Q. 14. (i) $x^2 = 176^2 + 135^2 - 2(176)(135) \cos 38.79°$

$= 12{,}161.66354$

$x = 110.28 \text{ km}$

$\cong 110 \text{ km}$

(ii) Journey = 176 + 110 + 135 = 421 km

$$T = \frac{D}{S} = \frac{421}{280} \cong 1.5 \text{ hrs.}$$

Total trip 1.5 + 1 + 2 = 4.5 hrs.

(iii) We need both journey times.

Car : $\dfrac{127}{50 \text{ km/h}}$ = 2.54 hrs = 2 hrs 32 mins

Helicopter : $\dfrac{110}{280 \text{ km/h}}$ = 0.39 hrs   Limerick to Waterford

$\dfrac{176}{280}$ = 0.63 hrs   Dublin to Limerick

Stopover   1 hr

Total time to Waterford including stopover = 2.02 hrs

= 2 hrs 1 min

The car arrives in Waterford 31 mins after the helicopter.

The helicopter must be in the air in 2 hrs from time of landing.

Max. length for meeting is 1 hr 29 mins.

(iv) This depends on the student's own personal response.

Q. 15. (i) $\cos B = \dfrac{33.07^2 + 33.07^2 - 35.42^2}{2(33.07)(33.07)}$

$B = \cos^{-1}(0.4264)$

$B = 64.76°$

(ii) $\dfrac{1}{2}$ (33.07)(33.07) sin 64.76°

= 494.61 m^2

(iii) 494.61 × 4 = 1978.44 m^2

(iv) 35.42 × 35.42 = 1254.5764 m^2

Q. 16. (i) $|AC|^2 = 35^2 + 40^2 - 2(35)(40) \cos 120°$

= 4225

$|AC|$ = 65 m

(ii) 35 + 40 + 65 = 140 m

(iii) Use area of Δ.

$\dfrac{1}{2}$ (35)(40) sin 120° = $\dfrac{1}{2}$ (35)(⊥height)

40 sin 120° = 34.64 metres

<u>or</u>

$\sin 60° = \dfrac{x}{40}$

$x = 40 \sin 60°$

$x = 34.64$ metres.
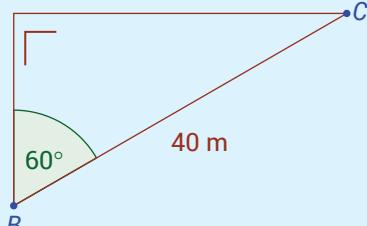
Exercise 17.9

Q. 1. (i) $\dfrac{50}{360}\pi(10)^2$

$= 13.89\pi\,\text{cm}^2$

(ii) $\dfrac{150}{360}\pi(12)^2$

$= 60\pi\,\text{cm}^2$

(iii) $\dfrac{210}{360}\pi(18)^2$

$= 189\pi\,\text{cm}^2$

Q. 2. (i) $2\pi(20)\left(\dfrac{40}{360}\right)$

$= 4.44\pi\,\text{cm}$

(ii) $2\pi(25)\left(\dfrac{100}{360}\right)$

$= 13.89\pi\,\text{cm}$

(iii) $2\pi(30)\left(\dfrac{170}{360}\right)$

$= 28.33\pi\,\text{cm}$

Q. 3. (i) $\dfrac{\theta}{360}\pi(6)^2 = 6\pi\,\text{cm}^2$

$\dfrac{6\theta}{360} = 1$

$\theta = 60°$

(ii) $\dfrac{\theta}{360}\pi(12)^2 = 54\pi$ (iii) $\dfrac{\theta}{360}\times 81\pi = 45\pi$

$\dfrac{\theta}{360} = \dfrac{54}{144}$ $\dfrac{\theta}{360} = \dfrac{45}{81}$

$\theta = 135°$ $\theta = 200°$

Q. 4. (i) $\dfrac{\theta}{360}\times 2\pi(18) = 7\pi$

$\dfrac{\theta}{360} = \dfrac{7}{36}$

$\theta = 70°$

(ii) $\dfrac{\theta}{360}\times 2\pi(6) = 4\pi$

$\dfrac{\theta}{360} = \dfrac{4}{12}$

$\theta = 120°$

Q. 5. Area of $\Delta\;\dfrac{1}{2}(13)(10)\sin 40° = 41.7812$

Sector $\dfrac{40}{360}\times \pi \times 15^2 = 78.5398$

Shaded $= 78.5398 - 41.7812$

$= 36.7586$

$\cong 36.8\,\text{m}^2$

Revision Exercises

Q. 1. (a) $\sin\alpha = \dfrac{20}{29}$ $\cos\alpha = \dfrac{21}{29}$ $\tan\alpha = \dfrac{20}{21}$

$\sin\beta = \dfrac{21}{29}$ $\cos\beta = \dfrac{20}{29}$ $\tan\beta = \dfrac{21}{20}$

(b) (i) $\dfrac{\sin\alpha}{7} = \dfrac{\sin 100°}{15}$

$\sin\alpha = \dfrac{7\sin 100°}{15}$

$\alpha = \sin^{-1}(0.4596)$

$\alpha = 27°$

(ii) $\dfrac{k}{\sin 53°} = \dfrac{15}{\sin 100°}$

$k = \dfrac{15\sin 53°}{\sin 100°}$

$k = 12.16\,\text{cm}$

(iii) $y^2 = 12.16^2 + 20^2 - 2(12.16)(20)\cos 120°$

$y^2 = 791.0656$

$y = 28.13\,\text{cm}$

Q. 2. (a)

$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
p	$-q$	$\dfrac{-p}{q}$

(b) (i) $\dfrac{1}{2}(x)(\sqrt{3})\sin 80° = 3$

$0.85290x = 3$

$x = 3.52$

(ii) $y^2 = 3.52^2 + \sqrt{3}^2 - 2(3.52)(\sqrt{3})\cos 80°$

$= 13.2730$

$y = 3.64$

Q. 3. (i) $\dfrac{x}{\sin 70°} = \dfrac{6}{\sin 65°}$

$x = \dfrac{6\sin 70°}{\sin 65°}$

$x = 6.22\,\text{cm}$

(ii) $\dfrac{1}{2}(6.22 + 5)(6)\sin 45°$

$= 23.8012$

$= 24\,\text{cm}^2$

Q. 4.

A	30°	45°	60°
sin A	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{3}}{2}$
cos A	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$
tan A	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$

(i) sin A = cos 60°

   sin A = $\dfrac{1}{2}$

   A = 30°

(ii) tan B = 1

   B = 45°

(iii) sin C = cos C

   C = 45°

(iv) sin D cos 30° = $\dfrac{3}{4}$

   sin D $\left(\dfrac{\sqrt{3}}{2}\right)$ = $\dfrac{3}{4}$

   sin D = $\dfrac{3}{4} \times \dfrac{2}{\sqrt{3}}$

   = $\dfrac{3}{2\sqrt{3}}$

   = $\dfrac{\sqrt{3}}{2}$

   D = 60°

Q. 5. (i) r = 2 cm

(ii) $\dfrac{60°}{360°} \times \pi \times 2^2$

   $\dfrac{2\pi}{3}$ cm^2

(iii) $\dfrac{1}{2}$(10)(10) sin 60°

   43.30 cm^2

(iv) $25\sqrt{3} - 2.09(2)$

   = 39.12 cm^2

Q. 6. (i)
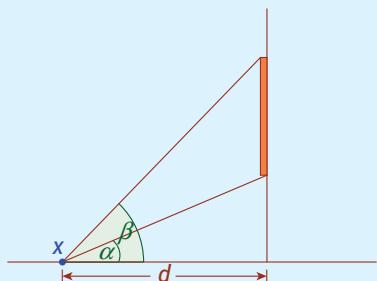
First, John should set up the clinometer at a point, x, a distance d from the base of the wall. The value of d can be found using the measuring tape. From x, John will measure the angle of elevation, β, of the billboard and the angle of elevation, α, of the bottom of the billboard.

(ii) $\dfrac{h}{5}$ = tan 23°

   h = 5 tan 23°

   h = 2.12 m = 212 cm

   $\dfrac{a + h}{5}$ = tan 45°

   $a + h$ = 5 tan 45°

   $a + h$ = 5

   a + 212 = 500

   a = 288 cm

(iii) height = $\dfrac{288}{8}$

   = 36 cm

Q. 7. (i) $x^2 = 6^2 + 10^2 - 2(6)(10) \cos 47$

   $x^2 = 54.1602$

   $x = 7.36$ m

(ii)

   tan 22 = $\dfrac{x}{6}$

   x = 6 tan 22

   x = 242 cm

(iii)
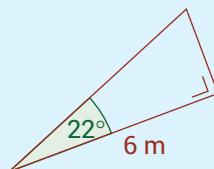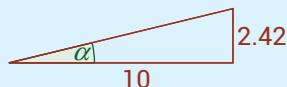

   tan $\alpha = \dfrac{2.42}{10}$

   $\alpha = \tan^{-1}(0.242)$

   $\alpha = 13.6040$

   $\cong 14°$

Q. 8. (i)
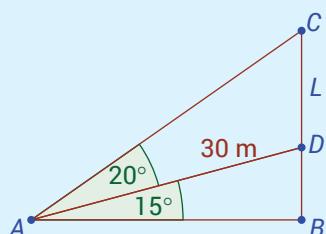
(ii) $\cos 15° = \dfrac{|AB|}{30}$

$|AB| = 30 \cos 15°$

$|AB| = 29$ m

$\tan 35° = \dfrac{|CB|}{29}$

$|CB| = 29 \tan 35° = 20$ m

Hill + tower = 20 m

Hill =

$\sin 15 = \dfrac{x}{30}$

$x = 8$ m

Tower = 20 – 8

= 13 m
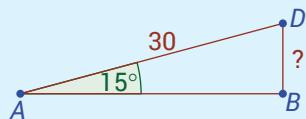
Exam Questions

Q. 1. (a) (i) $\dfrac{|BC|}{\sin 42°} = \dfrac{16}{\sin 110°}$

$\Rightarrow |BC| = \dfrac{16 \sin 42°}{\sin 110°}$

$= 11.393 = 11.39$ m

(ii) $|\angle DBC| = 180° – (42° + 110°) = 28°$

Area $\Delta = \dfrac{1}{2}ab \sin C$

$|\Delta BCD| = \dfrac{1}{2}(16)(11.39)\sin 28°$

$= 42.778 = 42.78$ m^2

(b) $|\angle BDA| = 180° – (63° + 42°) = 75°$

$a^2 = b^2 + c^2 – 2bc \cos A$

$|AB|^2 = 10^2 + 16^2 – 2(10)(16)\cos 75°$

$= 273.18$

$|AB| = 16.53$ m

Q. 2. (a) $\dfrac{1}{2}(8)(12)\sin 30° = 24$ cm^2

(b) $7^2 = 5^2 + 3^2 – 2(3)(5)\cos A$

$49 – 9 – 25 = –30 \cos A$

$\cos A = –\dfrac{1}{2}$

$A = 120°$

Q. 3. (a) (i) $|ED| = \sqrt{10^2 + 1.95^2} = 10.2$ m

$= 10.2$ m

(ii) $\cos |\angle AEB| = \dfrac{12}{14}$

$|\angle AEB| = \cos^{-1}\dfrac{12}{14}$

$|\angle AEB| = 31°$

(b) (i) $|\angle DEB| = 180 – (11 + 31)$

$|\angle DEB| = 180 – 42$

$|\angle DEB| = 138°$

(ii) $|DB|^2 = 14^2 + 10.2^2 – 2(14)(10.2)\cos 138°$

$|DB|^2 = 512.28216$

$|DB| = 22.6336$

$|DB| = 22.6$ m