**Number:** Operations – Multiplication 2

**5. Hidden in a Hexagon**

This task presents a puzzle to be solved. It includes opportunities to multiply decimal numbers by whole numbers. The puzzle encourages students to read clues, analyse them and plan an approach to finding a solution.

**Focus Skills:**

* *Applying and problem-solving*: Analyse problems and plan an approach to solving them.
* *Reasoning:* Reason systematically in a mathematical context.

**Teaching Points:**

* Ensure students understand the mathematical language used before beginning: ‘product’ is the result of multiplying numbers, ‘sum’ is the result of adding numbers and ‘difference’ is the result of subtracting one number from another.
* Encourage students to approach this task as they would a puzzle.
* The easiest clue to start with is not necessarily Clue A. Encourage students to read all the clues before they begin, analyse the clues and decide which one is the most useful to use first.
* Following initial exploration time, some students may benefit from scaffolding. This may mean telling them to focus on Clues C, D and E first.
* The multiplication element of the task can be solved using short or long multiplication. For example: If testing whether or not 12·3 × 9 × 8 = 590·4, students can multiply by 9 and then by 8, or multiply by 72. Using both methods would be an interesting way for students to verify their work.

**Anticipated Student Responses:**

|  |
| --- |
| **Task** |
| It is anticipated that students will begin with Clues C, D and E to determine the positions of the whole numbers, and that they will then use trial and improvement (the ‘Try It’ strategy) to determine the positions of the remaining two numbers. However, while less efficient, students could begin with Clues A and B and use a trial and improvement strategy to find possible solutions. Then they could use Clues C, D and E to check which solution is correct.1. As purple is double the value of blue, **purple must be 8** and **blue must be 4** as they are the only pair of numbers given in which one is double the other.
2. As the sum of blue and red is 10, and we now know that blue is 4, **red must be 6** as 4 + 6 = 10.
3. As red is $\frac{2}{3}$ the value of green, and we now know that red is 6, **green must be 9** as $\frac{2}{3}$ of 9 = 6.
4. As yellow × green × blue = 590·4, and we now know that blue is 4 and green is 9, by trialling the two decimal numbers we can determine that **yellow is 16·4**:

16·4 × 9 × 4 = 590·412·3 × 9 × 4 ≠ 553·51. By process of elimination, this means that **pink must be 12·3**. This can be verified using Clue B as we know that purple × pink × red = 590·4:

8 × 12·3 × 6 = 590·4**yellow = 16·4 green = 9 blue = 4 purple = 8 pink = 12·3 red = 6** |
| **Extension** |
| The extension activity encourages students to engage in further multiplication calculations:green × yellow × purple = 9 × 16·4 × 8 = 1180·8pink × red × blue = 12·3 × 6 × 4 = 295·2difference: 1180·8 – 295·2 = 885·6 |

**Algebra:** Equations

**17. The Marvellous Machine**

In this task, a machine that changes numbers is used to explore equations. Students are encouraged to write number sentences that include variables to solve each part of the task.

**Focus Skills:**

* *Integrating and connecting*: Represent mathematical ideas and processes in different modes: verbal, pictorial, diagrammatic and symbolic.
* *Reasoning*: Make hypotheses and carry out investigations to test them.
* *Reasoning*: Search for and investigate mathematical patterns and relationships.

**Teaching Points:**

* In the first part of the cartoon, we can see who has inspired Sinéad. Click each link for more information about [Pythagoras](https://kids.kiddle.co/Pythagoras), [Alicia Boole Stott](https://www.askaboutireland.ie/reading-room/life-society/science-technology/irish-scientists/alicia-boole-stott-%281860-/) and [William Rowan Hamilton](https://www.askaboutireland.ie/reading-room/life-society/science-technology/irish-scientists/william-rowan-hamilton-fr/index.xml).
* Provide different inputs and outputs based on simpler rules to explain the task, if necessary.

For example, 6 🡪 12, 7 🡪 14 and 50 🡪 100 (doubling/multiplying by 2).

* Encourage students to account for variables in their equations using frames or letters.

For example, [ ] × 20 = [ ] or a × 20 = b.

* Ask students to investigate what outputs would result from other inputs in each part of the task.
* For the extension task, encourage students to be creative with their equations.
* Having completed this task, students could be introduced to other function machines such as those available from [Top Marks](https://www.topmarks.co.uk/Flash.aspx?f=FunctionMachinev3).

**Anticipated Student Responses:**

There are multiple ways that students may express the rule in each part of the task. Below are some examples.

|  |  |  |
| --- | --- | --- |
| **Part A** | **Part B** | **Part C** |
| 8 × 20 = 16012 × 20 = 240**Rule:** a × 20 = bAlternative: a × 2 × 10 = b | 8 × 6 + 7 = 5512 × 6 + 7 = 7916 × 6 + 7 = 10324 × 6 + 7 = 151**Rule:** a × 6 + 7 = bAlternative: a × 3 × 2 + 7 = b | 82 ÷ 2 = 32 *or* (8 × 8) ÷ 2 = 32122 ÷ 2 = 72 *or* (12 × 12) ÷ 2 = 32162 ÷ 2 = 128 *or* (16 × 16) ÷ 2 = 128242 ÷ 2 = 288 *or* (24 × 24) ÷ 2 = 288**Rule:** a2 ÷ 2 = b *or* (a × a) ÷ 2 = bAlternative: a × (a ÷ 2) = b |

**Shape and Space:** Lines and Angles

**20. Penguin’s Maze**

This task requires students to use their computational thinking skills and understanding of angles to

problem-solve in a game-based context. They are encouraged to recognise angles in terms of rotation.

**Focus Skills:**

* *Integrating and connecting*: Represent mathematical ideas and processes in different modes: verbal, pictorial, diagrammatic and using manipulatives.
* *Communicating and expressing*: Listen to and discuss other students’ mathematical descriptions and explanations.

**Teaching Points:**

* In preparation for this task, ask students to stand up and face the front of the room. Instruct them to jump a certain number of degrees in a particular direction. For example: 90 degrees clockwise, 270 degrees anticlockwise. Focus on the quarter-turns (90 degrees, 180 degrees, 270 degrees, 360 degrees) and use a mixture of clockwise and anticlockwise directions.
* Ask students to follow the instructions written below the maze to ensure they understand how Penguin moves before writing instructions to help her reach the sea.
* Some students may find it difficult to visualise Penguin’s movements. To help them, place an interlocking cube on its side, using the top of the cube to represent Penguin’s beak.
* When creating their own maze in Part B, encourage students to incorporate different angle types.

**Anticipated Student Responses:**

|  |
| --- |
| **Part A**  |
| If Penguin follows the instructions given at the top of p. 44, she will end up in the square below the log on the bottom line of the grid: | Instructions to help Penguin reach the sea:Move two squares. 🡪Turn 270 degrees. 🡪 Move two squares. 🡪 Turn 90 degrees. 🡪 Move three squares. 🡪 Turn 270 degrees. 🡪 Move two squares. 🡪 Turn 90 degrees. 🡪 Move two squares. 🡪 Turn 270 degrees. 🡪 Move two squares. |

**Measures:** Length

**22. How Tall?**

This task encourages students to calculate estimate heights using appropriate metric units.

**Focus Skills:**

* *Integrating and connecting*: Recognise mathematics in the environment.
* *Applying and problem-solving*: Apply concepts and processes in a variety of contexts.
* *Reasoning*: Reason systematically in a mathematical context.

**Teaching Points:**

* Encourage the students to read Part A of the task, identify what information is presented and discuss what the question is asking them to do.
* Before they begin to work on the task, use a metre stick or measuring tape to mark 177 cm and 163 cm on the wall. Ask the students to predict what height they think Shane will be when he grows up and mark it on the wall.
* You could mark your own height on the wall, for students to compare in relation to the heights of Shane’s parents. Ask students to predict what height they are themselves. They could then measure each other’s heights and share this with the class.
* Parts A and B involve interpreting and applying a formula for finding the predicted adult heights of the students. Some students may need to see the formula being applied. Other heights could be used to demonstrate the formula in action. Calculators could also be made available to students. Encourage students to record their solutions in decimal form rather than with remainders. Use phrases such as: *We are dividing by 2, so what is half of 13 centimetres? That’s right, 12 and a half centimetres or twelve point five centimetres.*
* Part C encourages the students to work backwards. Ask questions, such as: *We have to work backwards now. In Part B, what was the last thing you did? What will we do first this time?* This may help students with their reasoning.
* As the students work backwards, they will first need to calculate the combined heights of Niamh’s grandparents. They will need to multiply 163 by two and add 13. This is the inverse of what the students did in Part B. Once they know the estimated combined height of Niamh’s grandparents, they can then use the second clue to estimate the height of her Gran and Grandad.
* Students could use an empty number line or a bar model to visualise the difference of 15 cm between Gran and Grandad.

Grandad’s height

339 cm

Gran’s height

15 cm

**Anticipated Student Responses:**

|  |
| --- |
| **Part A**  |
| Add parents’ heights: 177 cm + 163 cm = 340 cmAdd 13 cm: 340 cm + 13 cm = 353 cm Divide by 2: 353 cm ÷ 2 = 176·5 cm |
| **Part B** |
| Add parents’ heights: 177 cm + 163 cm = 340 cmSubtract 13 cm: 340 cm – 13 cm = 327 cm Divide by 2: 327 cm ÷ 2 = 163·5 cm |
| **Part C** |
| Students could work backwards using Mam’s height and inverse operation to multiply by 2: 163 cm × 2 = 326 cm Add 13 cm (for a girl) rather than subtract: 326 cm + 13 cm = 339 cm So the combined height of Gran and Grandad is 339 cm, but we know Grandad is 15 cm taller than Gran. Divide their combined height by 2: 339 cm $÷$ 2 = 169·5 cmAdd half of the 15 cm difference to find Grandad’s height: 169·5 cm + 7·5 cm = 177 cm Subtract half of the 15 cm difference to find Gran’s height: 169·5 cm – 7·5 cm = 162 cm |

**Data:** Representing and Interpreting Data 2

**29. Averages**

This task encourages students to explore and calculate averages of simple data sets.

**Focus Skills:**

* *Communicating and expressing:* Discuss and explain the processes used in solving a mathematical problem.
* *Integrating and connecting:* Understand the connections between mathematical procedures and concepts they use.

**Teaching Points:**

* Students will need to calculate the average of sets of numbers as well as work backwards to find which numbers result in a certain average. Ensure all students are clear on how to calculate averages (add the values to find the total and then divide by the number of values) to ensure this is not an initial barrier to accessing the problem.
* When the students move from Part B to Part C, they may still assume that the total number they are working with is 18. However, as we now have 4 numbers (not 3 numbers) the total they will be aiming at is 24. The same is true for part D.
* Encourage the students to explain how they solved the problem and how they can work backwards when they know the average of a certain group of numbers.
* Some students may benefit from working with 1–9 number cards to help them to organise their thinking.
* If students are finding this task difficult, encourage them to engage in trial and improvement, changing the numbers until they get an average of 6.

**Anticipated Student Responses:**

|  |  |
| --- | --- |
| **Part A** | **Part C** |
| Answers will vary. Possible answer: Pick 3, 4 and 5.3 + 4 + 5 = 12 12 $÷$ 3 = 4 The average of 3, 4 and 5 is 4. | Any four numbers that total 24 will result in an average of 6. For example:9 + 8 + 5 + 2 9 + 7 + 5 + 3 8 + 7 + 6 + 3 |
| **Part B** | **Part D** |
| Any three numbers that total 18 will result in an average of 6. For example: 9 + 4 + 59 + 8 + 16 + 9 + 3 3 + 7 + 8 | Any five numbers that total 30 will result in an average of 6. For example:9 + 8 + 7 + 5 + 18 + 7 + 6 + 5 + 49 + 8 + 7 + 4 + 2 |
| **Extension** |
| It is possible to get an average of 6 when you choose:* 6 numbers that total 36, i.e. 8 + 9 + 7 + 1 + 2 + 3.
* 7 numbers that total 42, i.e. 8 + 9 + 7 + 6 + 5 + 4 + 3

It is not possible to get an average of 6 with eight or nine numbers, unless you use the same number more than once.  |